



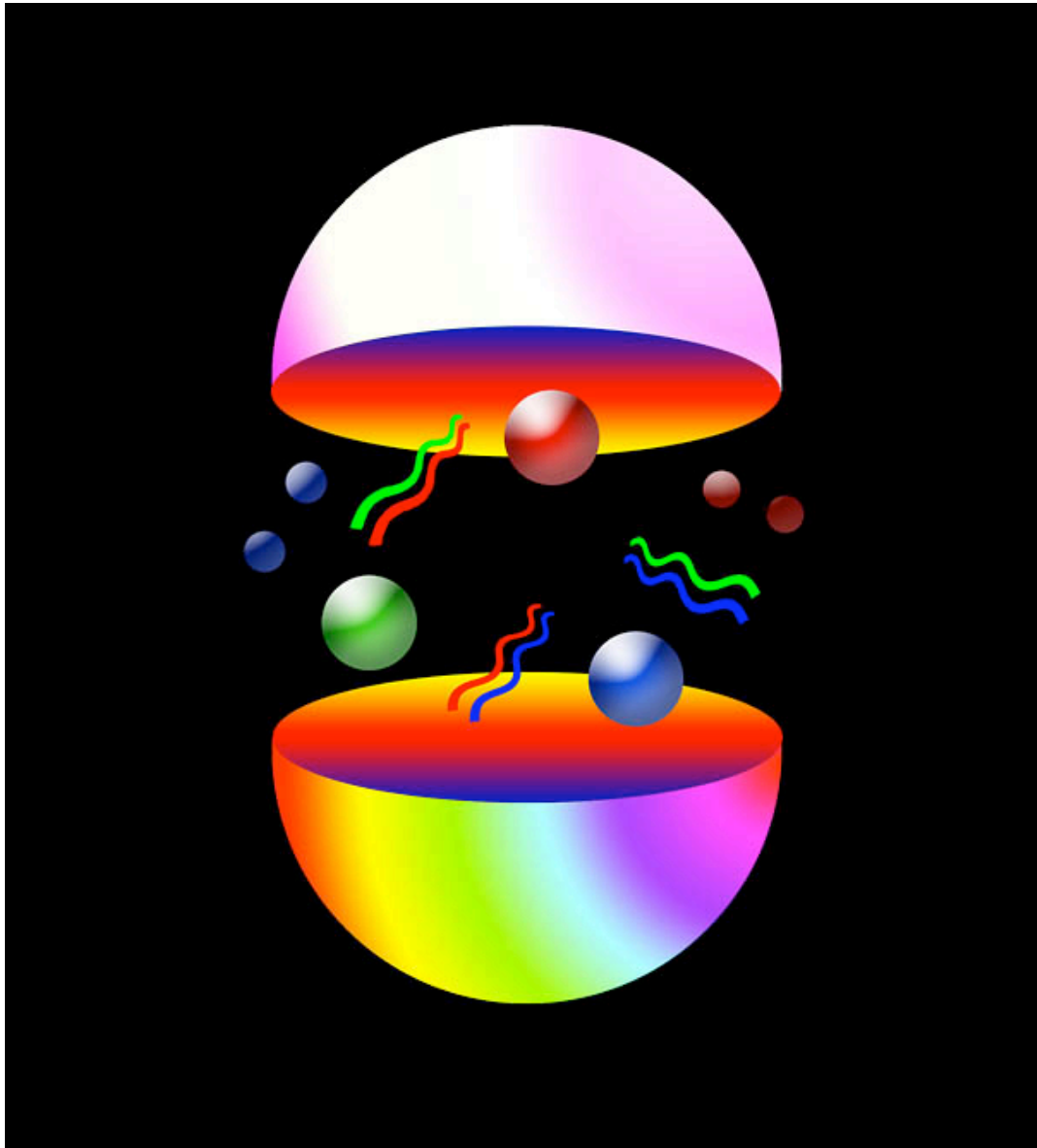
Lattice QCD

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BEAUTY 2006, Oxford, September 2006

Testing the Standard Model in the quark sector

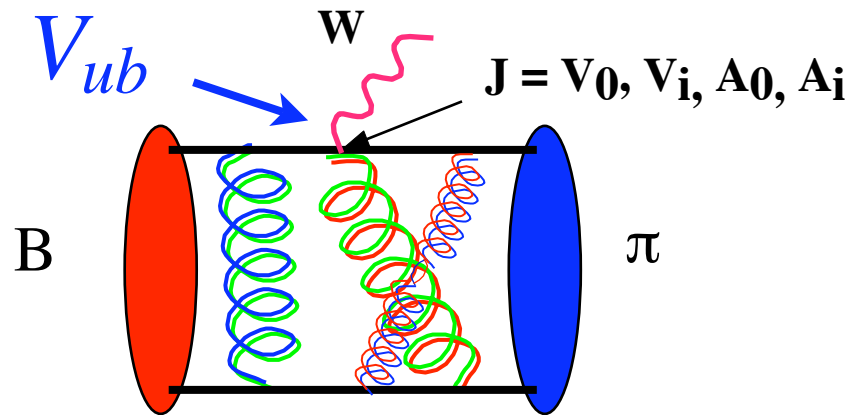
Problem: confinement



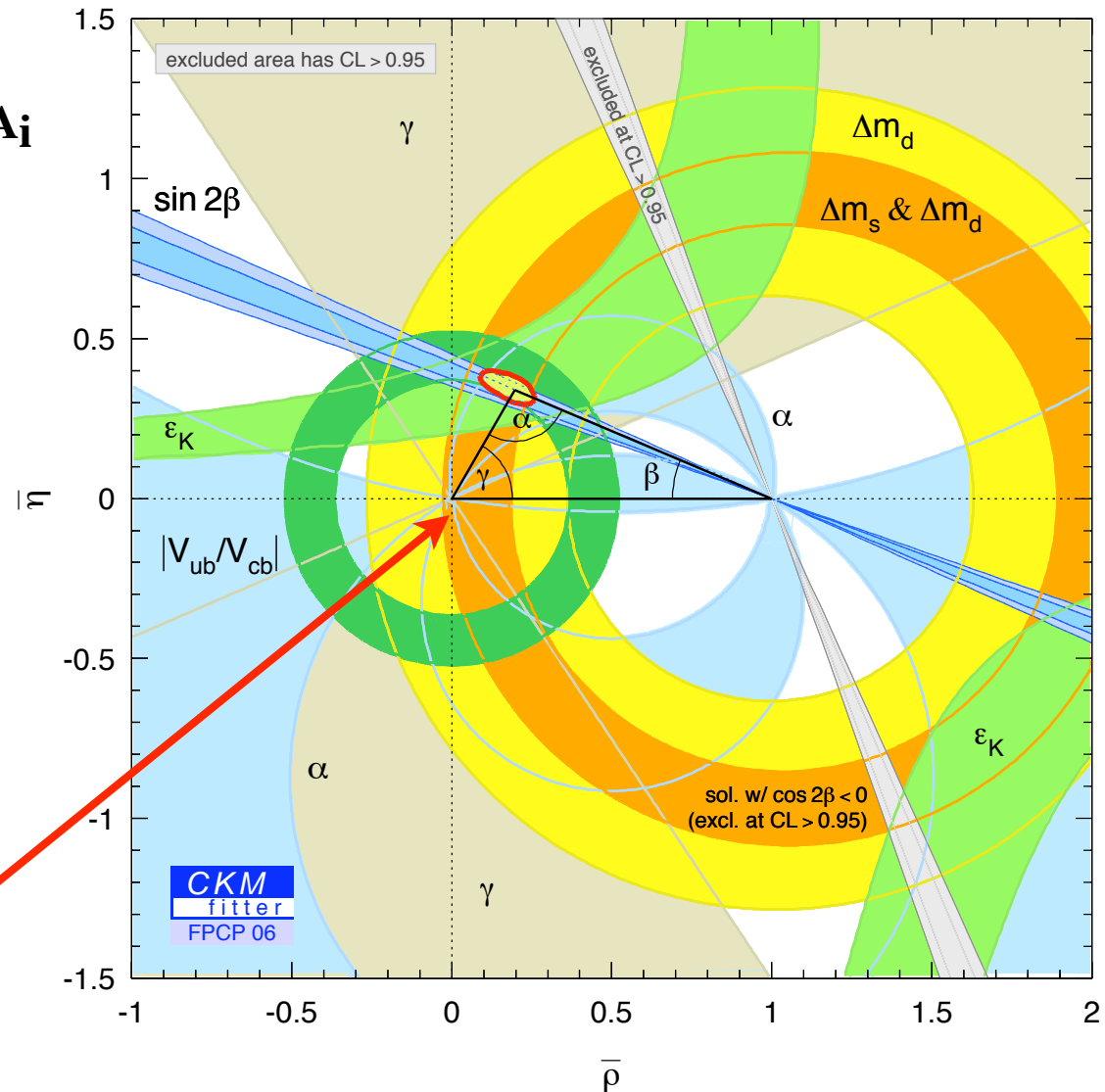
Need accurate expt on hadrons *and* theoretical calculations relating that back to quark properties.

QCD is theory of strong force - hard to calculate because strongly-coupled and nonlinear - needs numerical simulation. This is **lattice QCD**.

Focus : Determination of CKM matrix

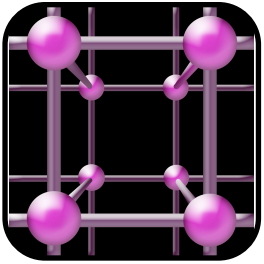


weak decay of quark
inside a hadron -
calculate in lattice QCD
 $\text{expt} = \text{CKM} \times \text{lattice}$



unitarity triangle sides

Precision (3%) lattice QCD needed

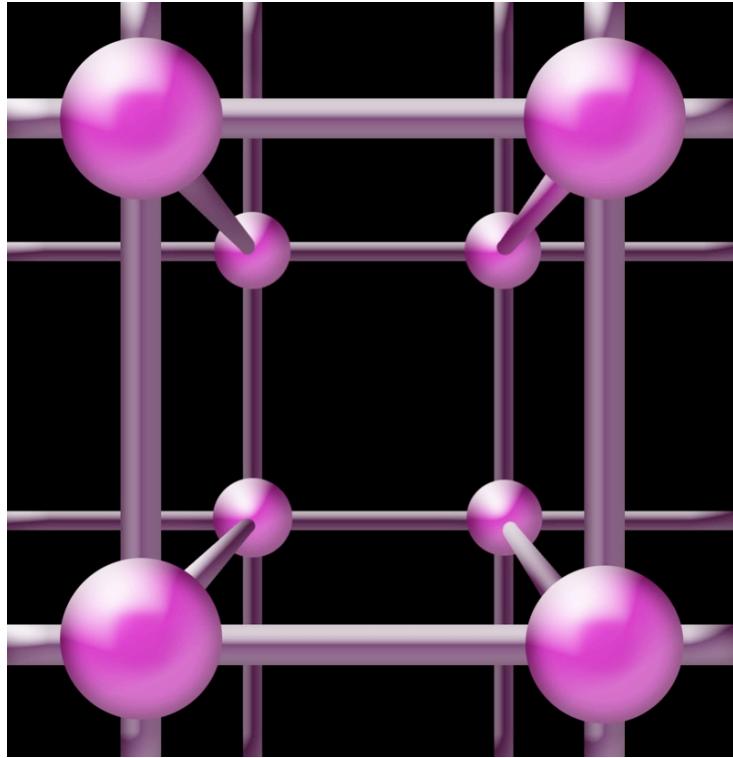


Take-home message

- There has been a revolution in the numerical simulation of the theory of the strong force (lattice QCD) since 2003
- Lattice QCD now delivering results : hadron masses that agree with expt; precise parameters of QCD; progress on decay rates needed to determine the CKM matrix accurately.

See: <http://www.physics.arizona.edu/lattice06/>

Lattice QCD



- Solve QCD by numerical evaluation of path integral:

$$\int dA_\mu d\psi d\bar{\psi} e^{-S_{QCD}}$$

- make integral finite with a space-time lattice
- Importance sampling - make gluon configs - ‘snapshots of vacuum’ and propagate quarks through them.
- ‘Measure’ e.g. hadron correlators on the gluon configs to calc. hadron masses and weak decay rates

Handling light quarks is a big headache

$$L_{q,QCD} = \bar{\Psi}(\gamma \cdot D + m)\Psi \equiv \bar{\Psi}M\Psi$$

For valence quarks, need to calc. M^{-1}

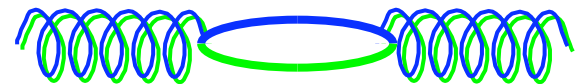
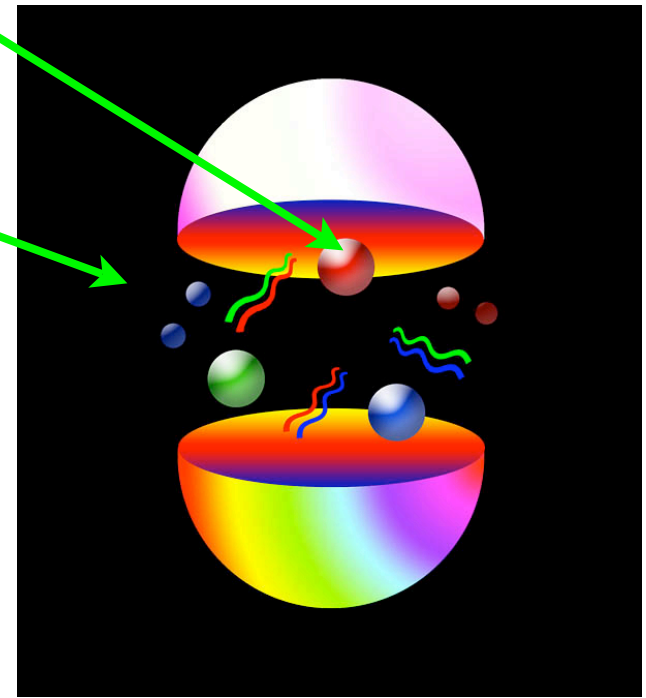
For sea quarks need to inc. $\det(M)$
in making gluon configs

Very costly as $m_q \rightarrow 0$

Early calcs:

Quenched Approximation - omitted sea quarks. This is not good enough for precision required.

Need to **unquench** with real s and light u/d - computing cost high



Issues for full lattice QCD: Disc. errors and chiral symmetry
- proliferation of light quark actions

Sharpe, LAT06

MILC: improved staggered, fast, 'ugly' fourth root of det.

$a = 0.06 - 0.18\text{fm}, m_{u/d}/m_s = 0.1, 0.2, 0.4, L = 2.5 - 3.0\text{fm}$

Analyses from **FNAL HPQCD, MILC, UKQCD** using c, b
valence quarks with Fermilab/NRQCD

RBC/UKQCD: Domain wall, slow

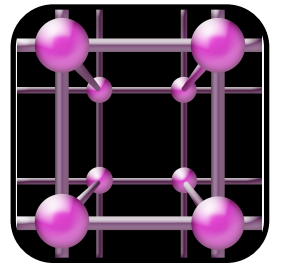
$a = 0.12\text{fm}, m_{u/d}/m_s = 0.2, 0.3, 0.4, L = 2.5 - 3.0\text{fm}$

PACS-CS: NP clover Wilson, new algorithms speed up

JLQCD: overlap, slow, but have 50 Tflops computer!

ETMC: twisted mass, faster than clover (?)

Lattice QCD results



MILC collaboration gluon configurations have:

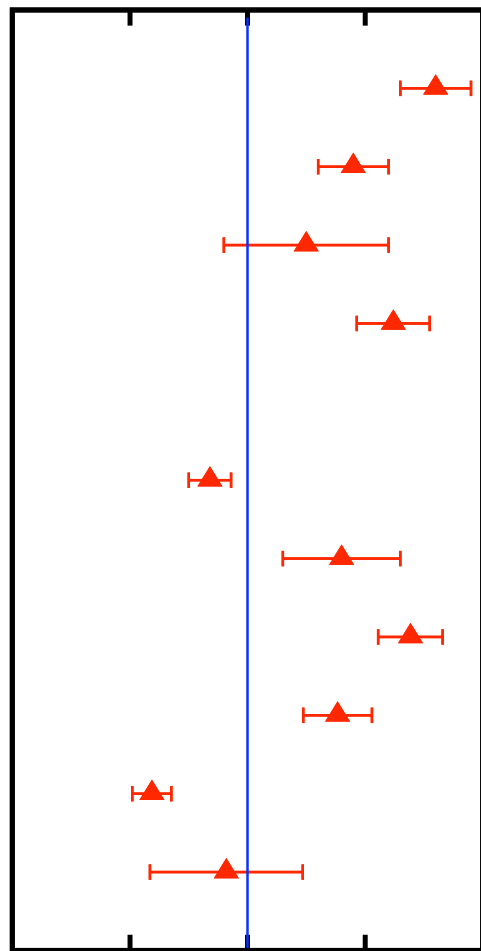
- 2+1 flavors of sea quarks, down to $m_{u/d} = m_s/10$.
- Many $m_{u/d}$ values; 2 m_s values
- 3 values of lattice spacing: 0.18fm, 0.12fm and 0.09fm
- Spatial volume exceeding $(2.5\text{fm})^3$ so lattice size $28^3 \times 96$

QCD has 5 parameters : 4 quark masses and a bare coupling. Must fix these using ‘gold-plated’ (i.e. stable) hadron masses. FNAL/HPQCD/MILC/UKQCD analysis of MILC configs. Fix: $a : M_{\Upsilon'} - M_{\Upsilon}$

$$m_{u/d} : M_{\pi} \quad m_s : M_K \quad m_c : M_{D_s} \quad m_b : M_{\Upsilon}$$

Calculate other gold-plated hadron masses as test. Results that follow are from this analysis.

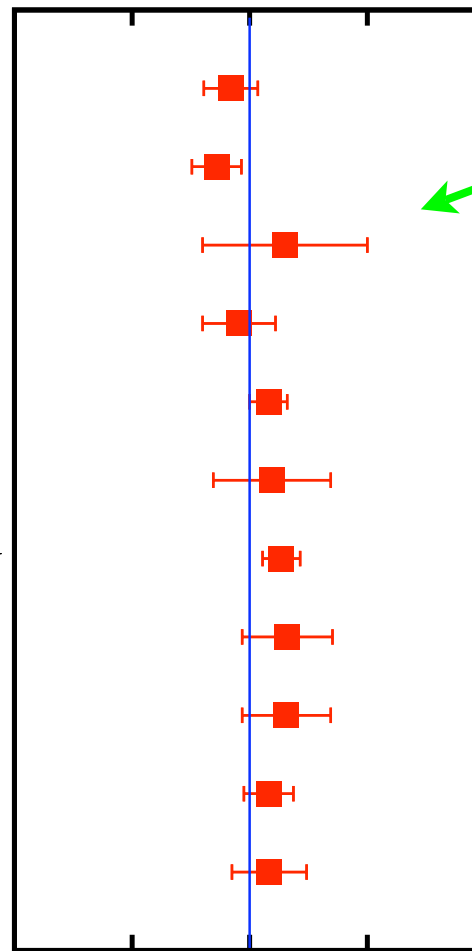
Summary of results - LQCD/expt - 2006 update



0.9 1 1.1

Quenched

f_π
 f_K
 $3m_\Xi - m_N$
 m_Ω
 $2m_{D_s} - m_{\eta_c}$
 $\psi(1P-1S)$
 $2m_{B_{s,av}} - m_Y$
 $Y(3S-1S)$
 $Y(2P-1S)$
 $Y(1P-1S)$
 $Y(1D-1S)$



0.9 1 1.1

$n_f=2+1$

Results including u,d and s sea quarks agree with experiment *across the board* -from light to heavy hadrons. Parameters of QCD are *unambiguous*

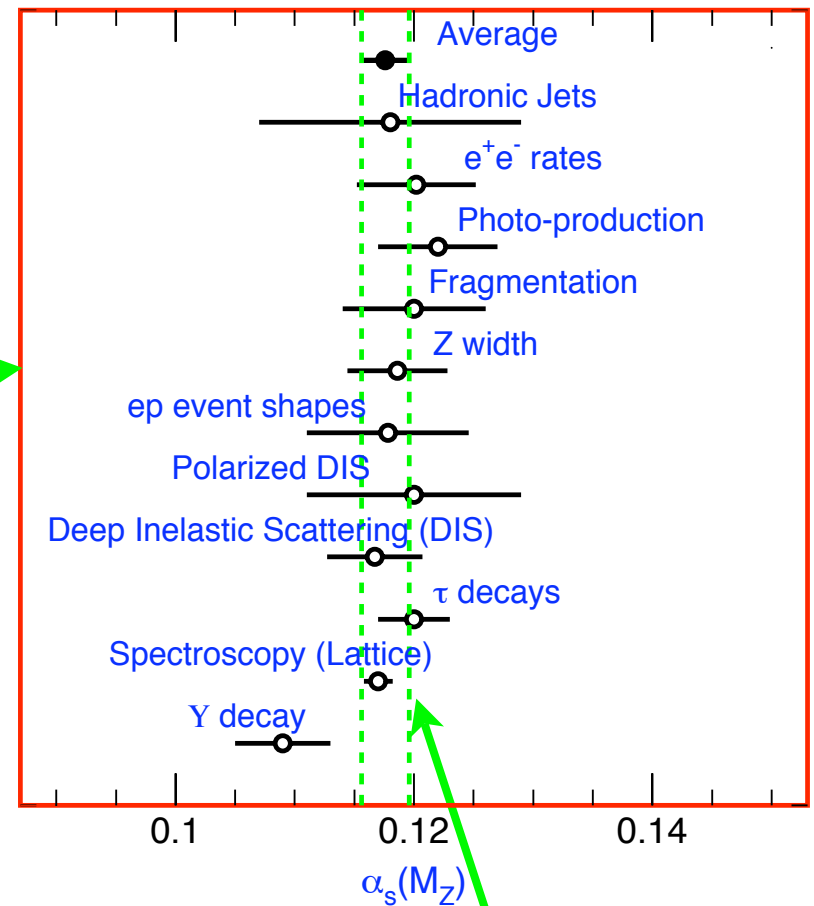
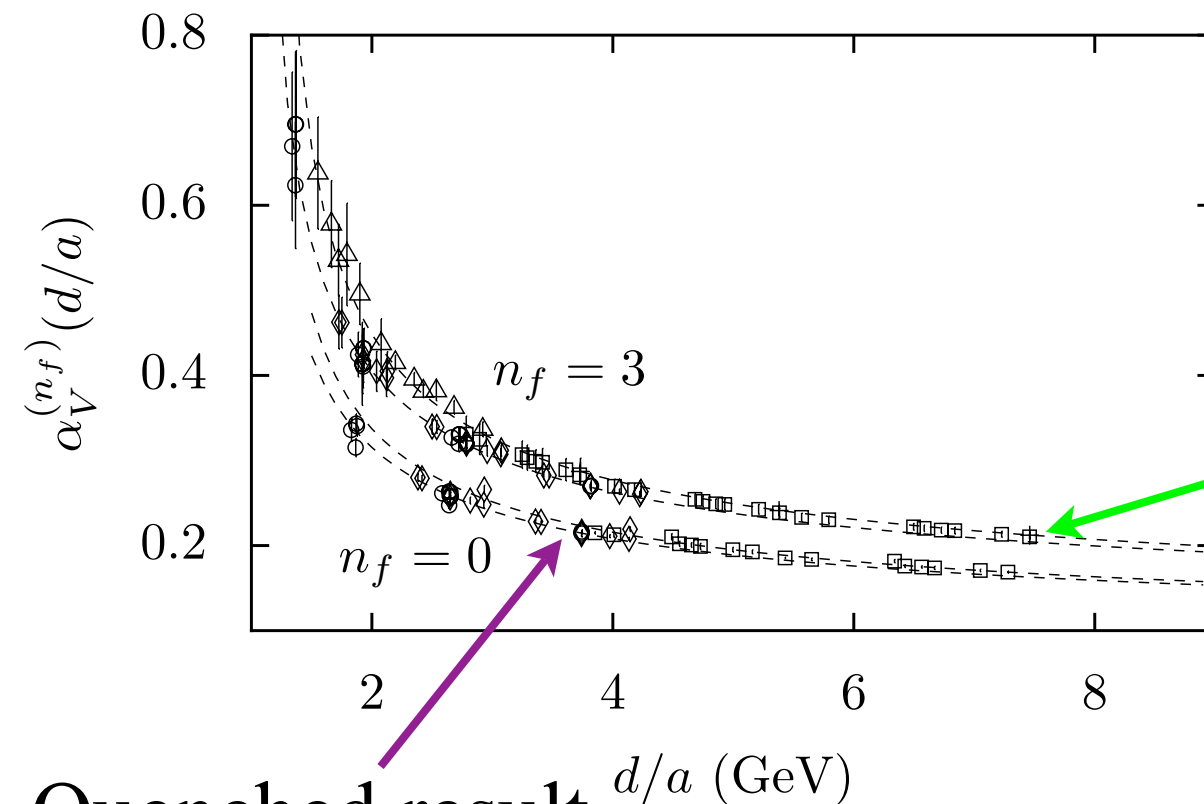
Quenched results are *wrong* and *ambiguous*

Determining QCD parameters:

$$\alpha_{\overline{MS}}^{(5)}(M_Z) = 0.1170(12)$$

2006 PDG

world av.= 0.1176(20)



Lattice QCD result
with sea quarks

Uses QCD pert. th. on lattice
for small gluon loops vs result
calc on lattice.

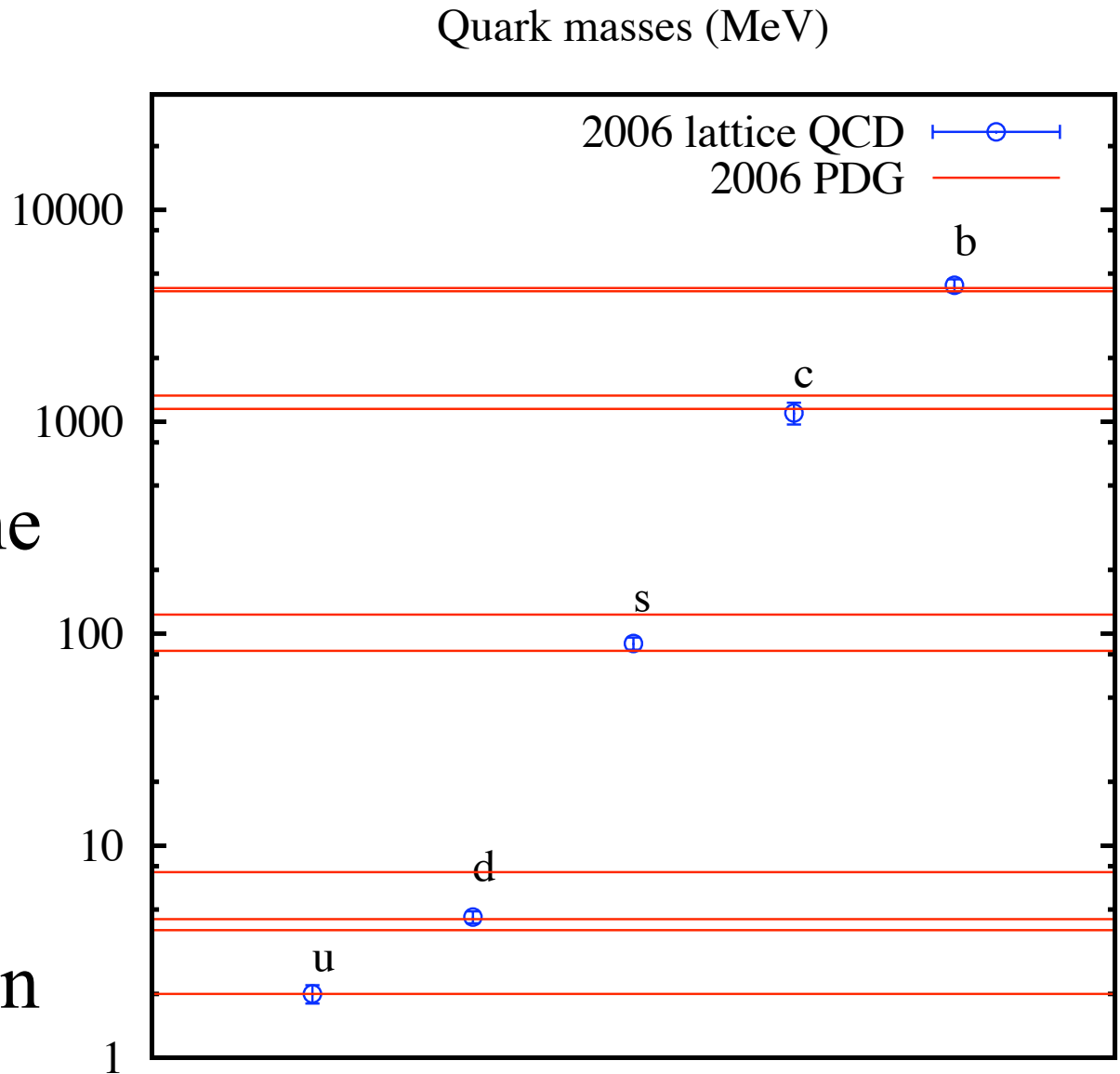
HPQCD, Mason et al, hep-lat/0503005

Determining parameters of QCD: m_q

Quarks never free so
cannot measure mass
directly

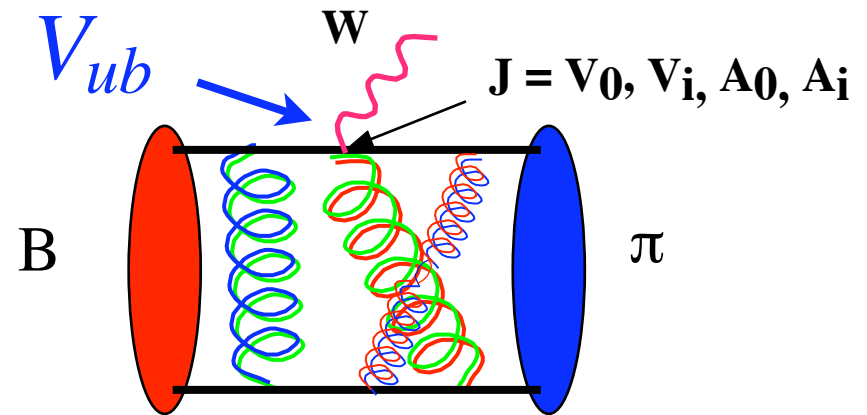
Masses are
parameters in lattice
QCD action - determine
by getting hadrons
masses right.

Convert to continuum
 \overline{MS} mass by QCD
pert. th. on lattice (main
source of error).



Weak decay rates and the CKM matrix

$$\left(\begin{array}{ccc}
 V_{ud} & V_{us} & V_{ub} \\
 \pi \rightarrow l\nu & K \rightarrow l\nu & B \rightarrow \pi l\nu \\
 & K \rightarrow \pi l\nu & \\
 V_{cd} & V_{cs} & V_{cb} \\
 D \rightarrow l\nu & D_s \rightarrow l\nu & B \rightarrow D l\nu \\
 D \rightarrow \pi l\nu & D \rightarrow K l\nu & \\
 V_{td} & V_{ts} & V_{tb} \\
 \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle &
 \end{array} \right)$$



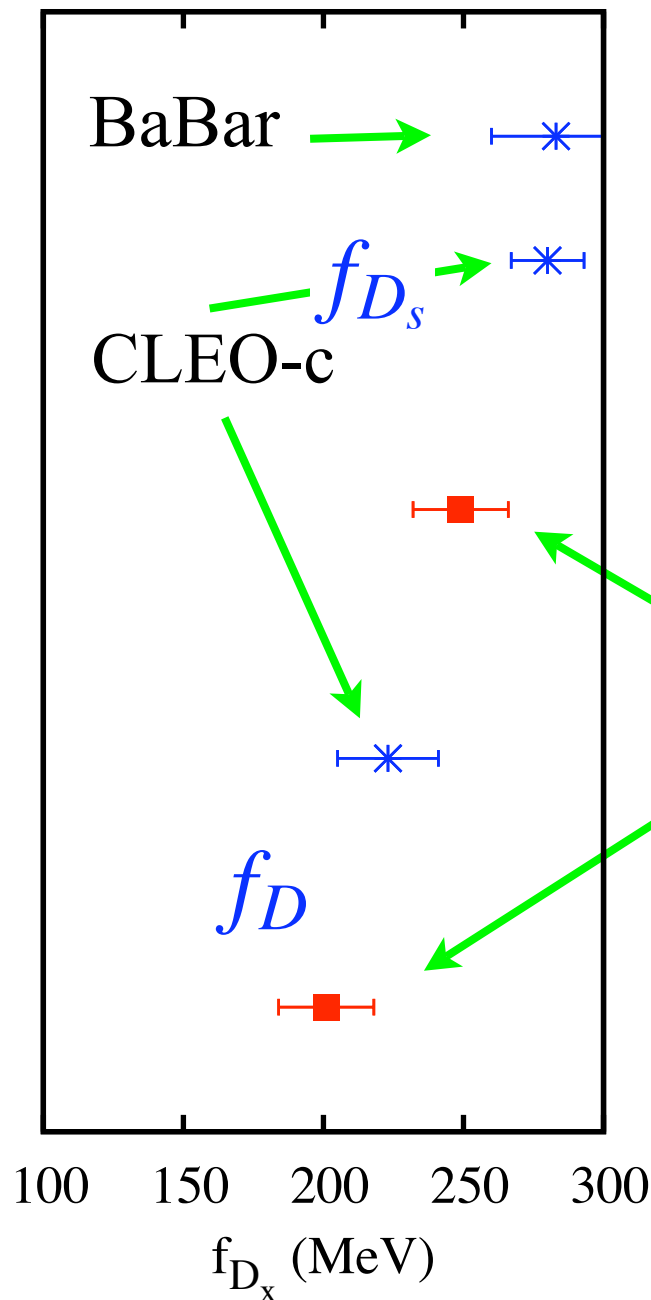
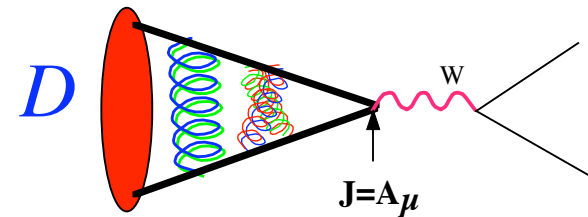
Lattice QCD calc. gives rate of basic weak decay from one *hadron to another*. CKM and lepton kinematics outside calc. Must ‘match’ lattice to continuum

Lattice QCD can calculate decay rates for at most one ‘gold-plated’ hadron in final state.

Possible for almost every element of CKM matrix.

* Need multiple cross-checks of lattice calcs in different systems e.g. Υ, B, D, ψ etc

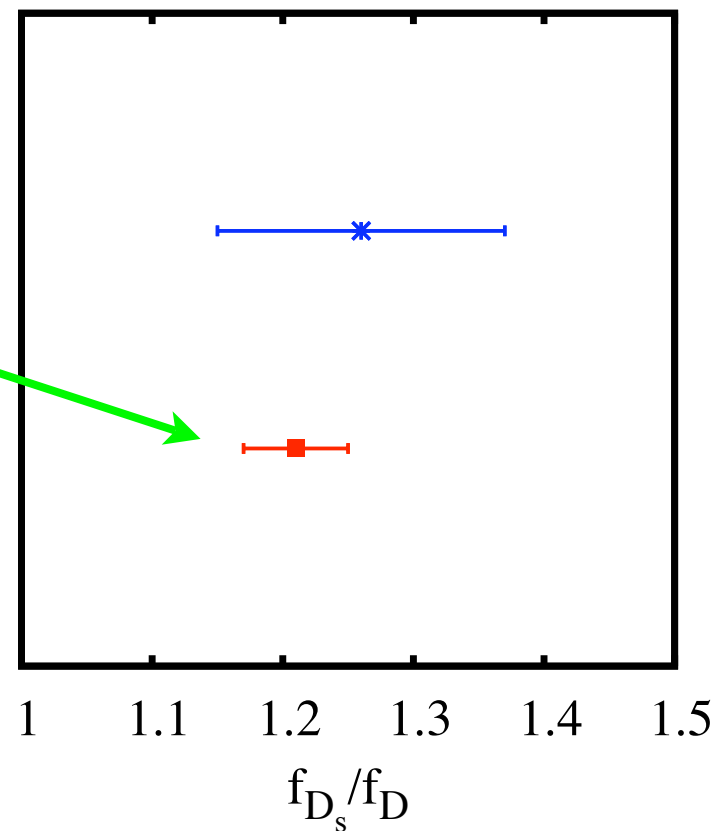
D meson leptonic decay rates



Test lattice QCD for **B** by *predns* for **D**.
 CLEO measure *lept. decay rate* of D/D_s.
 Convert to f_{D_x} given V_{cx}
 Lattice - direct
 calc. of f_{D_x}

Lattice QCD -
 FNAL/MILC
 10% errors th+expt -
 ratio 4% on lattice

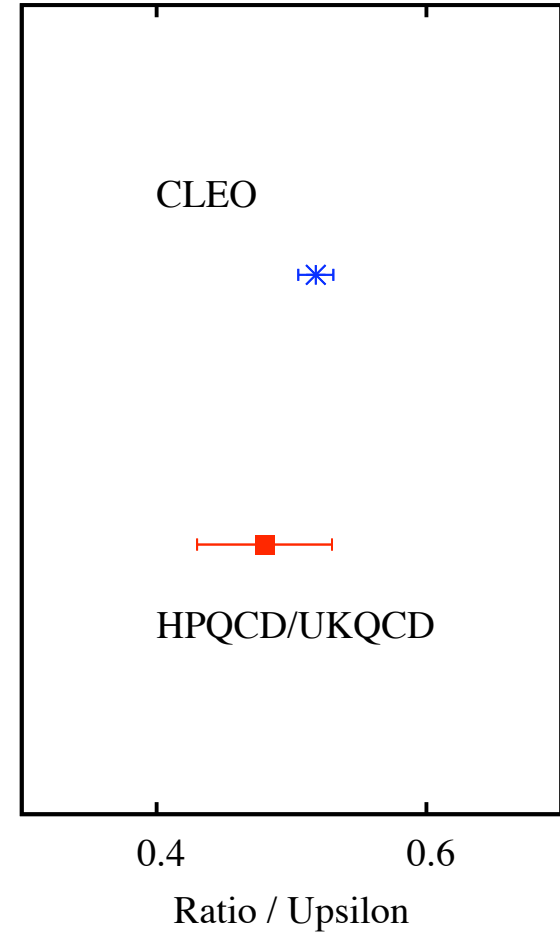
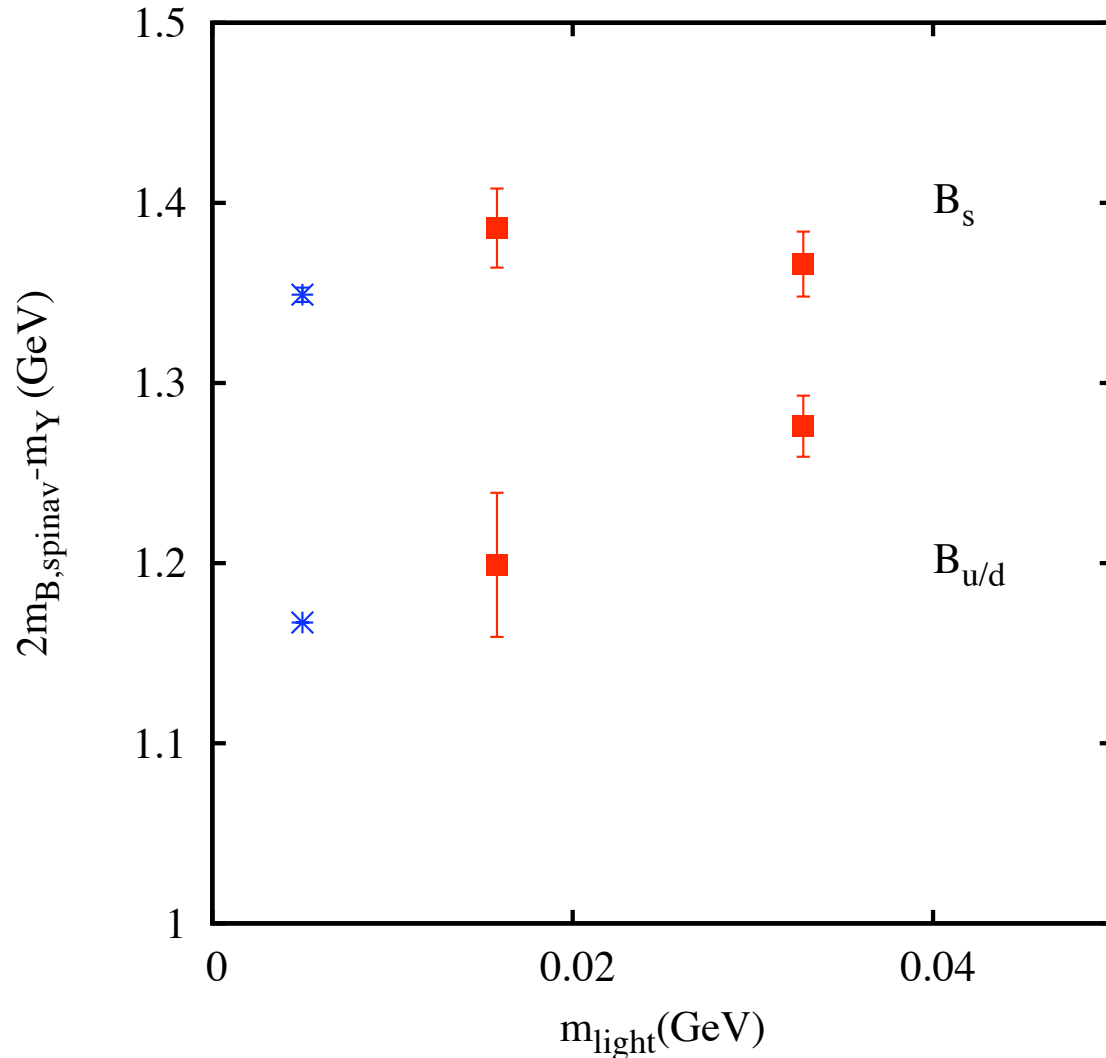
Lattice errors of
 few % possible



Direct tests of b quark physics in lattice QCD

Using NRQCD action for b quark, fix m_b from m_Y

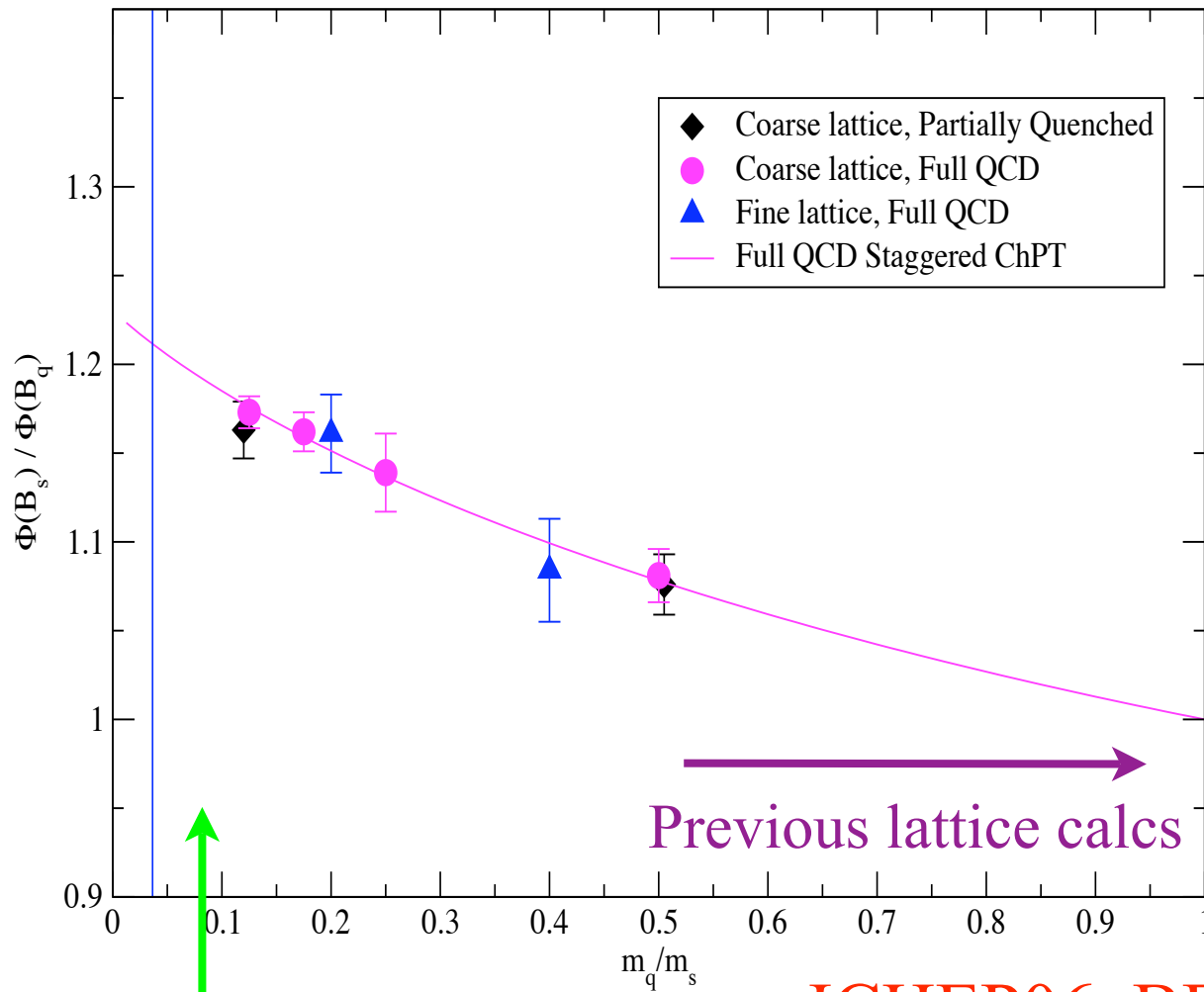
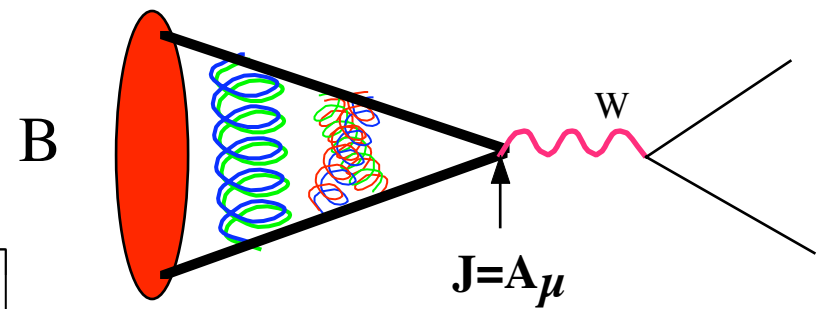
No free parameters left - test Y spectrum; B mass; $\Gamma(Y \rightarrow e^+e^-)$



HPQCD, hep-lat/0507013, CLEO, hep-ex/0512046

$$\frac{\Gamma_{ee}^{(2S)} M_{2S}^2}{\Gamma_{ee}^{(1S)} M_{1S}^2} = \frac{\langle Y' | J_{em} | 0 \rangle^2}{\langle Y | J_{em} | 0 \rangle^2}$$

B meson leptonic decays



log dependence on
light quark mass

ICHEP06, BELLE:

$\text{Br}(B \rightarrow \tau \nu)$ with HFAG V_{ub}

$$f_B = 229(36)(34)\text{MeV}$$

Lattice QCD results
using NRQCD b quarks
on MILC configs:

$$f_B = 216(22)\text{MeV}$$

$$\frac{f_{B_s}}{f_B} = 1.20(3)$$

Main lattice error
from matching.
Need α_s^2 calc
Cancels in ratio.

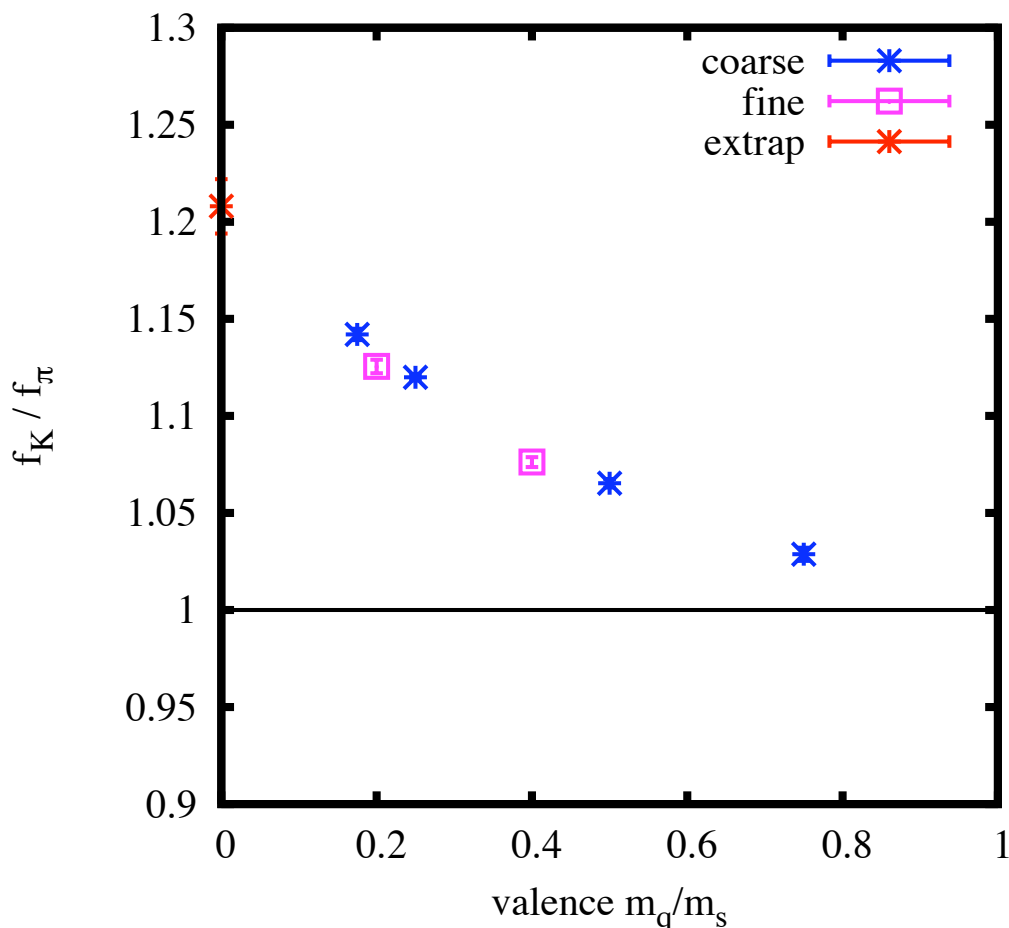
HPQCD, Gray et al,
hep-lat/0507015;

Can we do better ?

Try double ratio to f_K/f_π

Becirevic et al, hep-ph/0211271

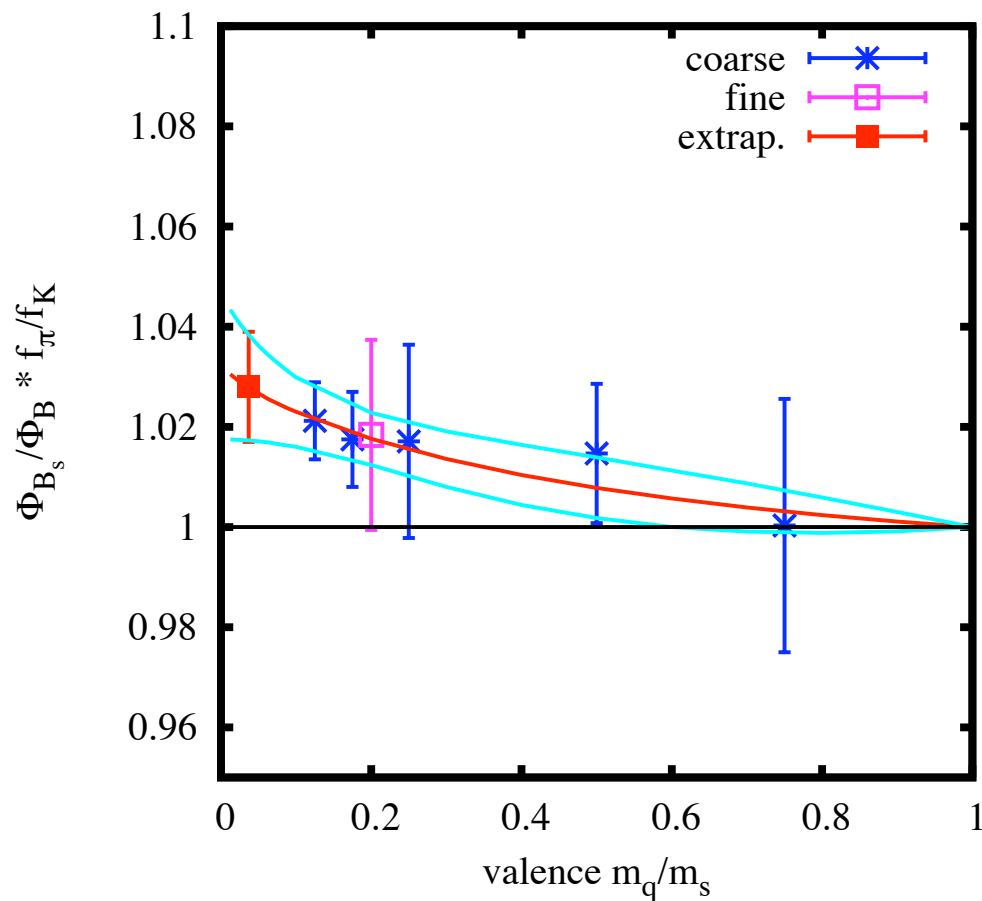
ratio decay constants MILC configs



MILC results - $f_K/f_\pi = 1.208(2) \left(\begin{smallmatrix} +7 \\ -14 \end{smallmatrix} \right)$
 yield $V_{us} = 0.2223(26)$

Competitive with PDG from SL decay
 Sugar, MILC, LAT06

double ratio decay constants MILC configs



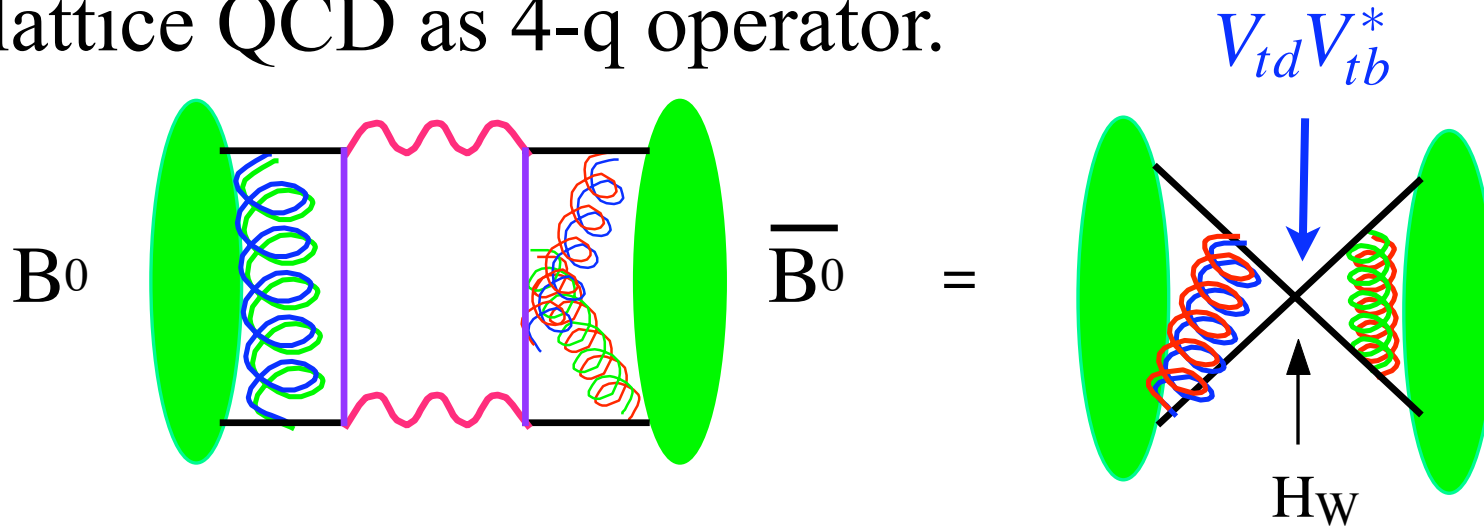
Much flatter chiral extrapoln

$$f_{B_s}/f_B \times f_\pi/f_K = 1.019(11)$$

f_{B_s}/f_B Total error:
 HPQCD, 2%

Why do we want this ratio? Neutral B mixing

B/B_s oscillation rate determined by box diagram. Calculate in lattice QCD as 4-q operator.



Parameterise with $f_B^2 B_B$ where f_B is decay constant.

$$\Delta M_x = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tx}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_x} f_{B_x}^2 \hat{B}_{B_x}$$

Ratio really required is $\xi = \frac{f_{B_s} \sqrt{B_{B_s}}}{f_B \sqrt{B_B}} \longrightarrow \left| \frac{V_{td}}{V_{ts}} \right| = \xi \sqrt{\frac{\Delta M_d M_{B_s}}{\Delta M_s M_{B_d}}}$

ξ used in CKM fits mixes lattice calcs and has 5% error

EXPECT ξ from full lattice QCD with 3% errors in 2007.

NEW lattice QCD result for B_s mixing matrix element

$$f_{B_s} \sqrt{\hat{B}_{B_s}} = 0.281(21) \text{ GeV} \quad \text{Shigemitsu, HPQCD, LAT06}$$

PRELIMINARY

gives $\Delta M_{B_s} = 20(3) ps^{-1}$ using unitarity $|V_{ts}^* V_{tb}|$

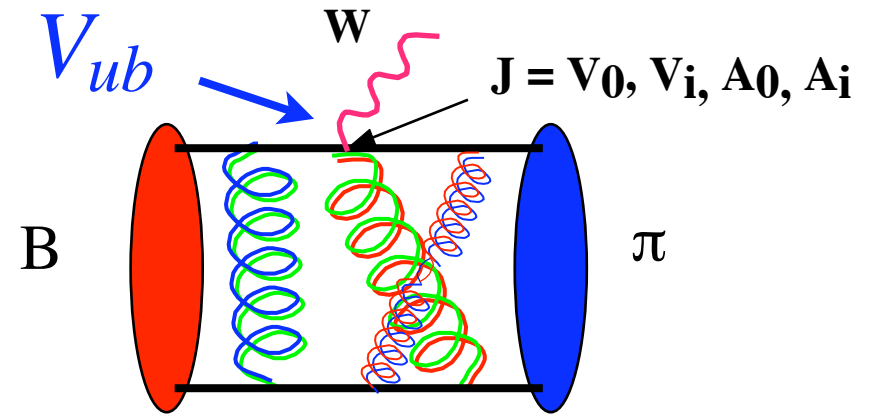
cf CDF $\Delta M_{B_s} = 17.3(3) ps^{-1}$

or, use expt+lattice to extract CKM elements:

$$|V_{ts}^* V_{tb}| = 3.8(3)(1) \times 10^{-2}$$

B semileptonic form factors

$B \rightarrow \pi l \nu$ and V_{ub}



HFAG06 + LQCD f.f. :

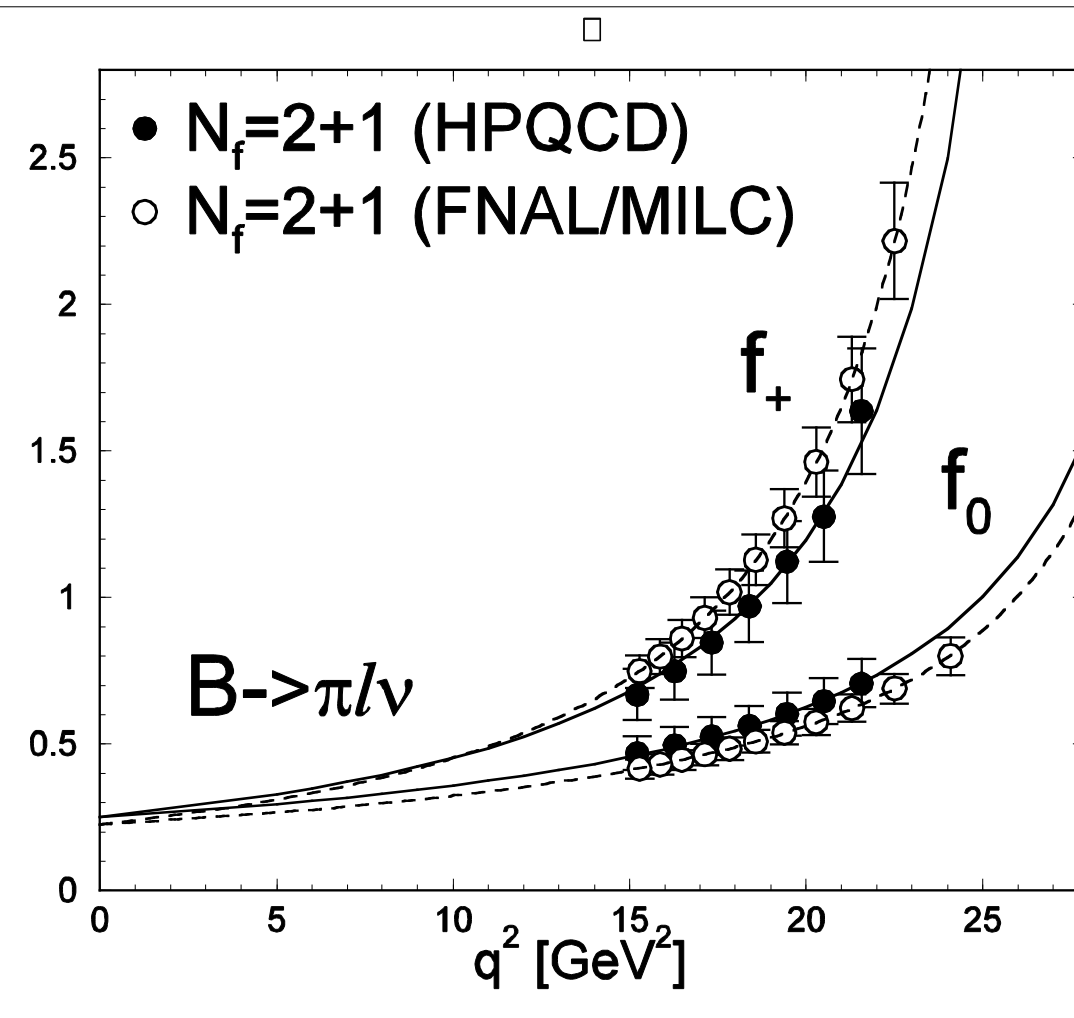
$$V_{ub} \times 10^{-3} = 3.93(26)(41)$$

HPQCD 7% expt
12% lattice

$$= 3.54(23)(61)$$

FNAL/MILC

Need to improve lattice
calcs - extend q^2 range
and match to contnm
more accurately



HPQCD. hep-lat/0601021; FNAL/MILC, hep-lat/040830;
Belle, BaBar, CLEO, ICHEP06, L. Gibbons here

Conclusions

- Accurate Lattice QCD calculations are now maturing. Tests of hadron masses and determination of parameters of QCD are at the few % level.

Future

- 10% errors on decay matrix elements for CKM. Beat down errors further on B and D physics for impact on unitarity triangle.
- Longer term - work on harder calculations e.g. for proton structure and unstable particles.
- Calculations beginning now that use other quark formalisms that are numerically more expensive.